Water Resources Management **16:** 1–22, 2002.

Effect of Averaging Operators in Fuzzy Optimization of Reservoir Operation

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(Received: 6 June 2001; accepted: 14 December 2001)

Abstract. Fuzzy multiobjective decision making models generally rely on the aggregation of the objectives to form a decision function. The generalized averaging operator is usually adopted for aggregating multiple and unequal objectives because it allows trade-off amongst the objectives, and has been shown to be suitable to model human decision making behavior. In the field of water resource management, most of the decision-making problems involving the generalized averaging operator implicitly assume the decision maker (DM) is rather optimistic. The analysis of the DM's behavior during the aggregation process and its impact on the performance of the system, has therefore never been addressed by many researchers and decision makers. The aim of this paper is to investigate the relationship between decision makers' index of optimism and the long-term performance of a reservoir resource. More specifically, the generalized averaging operator, whose parameter can be interpreted as the DM's index of optimism, is imbedded into a fuzzy stochastic dynamic program (FSDP). This approach is developed and implemented to derive optimal operating policies for the hydroelectric complex of the Uruguay River basin in Southern Brazil. FSDP-derived policies with different indices of optimism are then compared with simulation. We show that system performance may be influenced by the decision maker's behavior during the aggregation, and that the optimistic assumption may not yield to satisfactory results, especially during critical time periods.

Key words: flexible mathematical programming, fuzzy set, multiobjective decision-making, reservoir operation

1. Introduction

Hydroelectric reservoir operation involves stochastic input (the natural inflows), nonlinear system dynamics (the release decision depends on the product of the release and the storage level), feedbacks (immediate and future consequences are interdependent), and imprecise secondary operating objectives (environmental consideration, flood control, etc.). Dynamic programming (DP) is an optimization approach that is well suited to hydropower systems. Stochastic DP (SDP) formulations are traditionally adopted to deal with the stochastic nature of the inflows. Yeh (1985) provides a state-of-the-art review of the applications of DP and SDP in reservoir optimization problems. Example of applications of explicit SDP models can be found in Stedinger *et al.* (1984), Kelman *et al.* (1990), Tejada-Guibert *et al.* (1993). An SDP formulation is used by Pereira (1989) to derive optimal operating strategies of Brazil's hydrothermal system. In the SDP model, release decisions are given in each time period as a function of both the storage level at the beginning of the time period, and the hydrologic state variable(s). It also provides a cost-to-go function, which represents the value of the system over some period given the system's status, i.e. the volume in storage and the hydrologic condition.

Traditionally, SDP models rely on the use of a penalty function that represents the economic losses incurred from deviations from release and/or storage targets. In particular, when dealing with hydroelectric reservoirs, the flood control objective is usually apprehended by penalizing revenues from energy generation when the operation leads to unsafe storage level and/or release. The main issue associated with the use of penalties comes from the fact that they are arbitrarily selected by the reservoir operators, and therefore are highly subjective (Teegavarapu and Simonovic, 1999). As a matter of fact, the shape of the penalty function is often imposed by the decision makers (DM) to reflect a policy designed deliberately to achieve specific results (Datta and Burges, 1984). This management practice is now subject to critics, mainly because it fails to efficiently recognize multiple objectives, and suffers from a lack of transparency and participation since it only involves few stakeholders. Yet, sustainable management of water resources recommends that particular attention be paid to these issues (Hjorth et al., 1998; Loucks 2000; Nicklow, 2000).

The fuzzy SDP (FSDP) approach proposed by Tilmant *et al.* (2001) addresses these issues by providing a multiparticipatory framework in which both qualitative and quantitative objectives can be considered. For example, the *efficient production of hydroelectricity* and the creation of *adequate flood control space* are handled through the concept of fuzzy sets. The motivation for using fuzzy sets is two-fold. First, from a theoretical point-of-view, it allows us to exploit the theory of fuzzy sets and fuzzy logic to deal with the imprecision (vagueness) that characterizes many components of the decision-making process in the water resources area. Secondly, from a practical point-of-view, this approach does not require the implementation of controversial economic valuation techniques, such as the contingent valuation method, to monetize intangibles (Shabman and Stephenson, 2000). Rather, the FSDP approach relies on surveys of water users and managers to capture their preferences and expectation with respect to certain objectives, whether they are classic economic ones such as hydropower generation, or inherently vague ones such as the minimum flow requirement for maintaining *suitable fish habitat*. In the FSDP approach, the solution of the decision making problem still relies on the Bellman and Zadeh's framework, which consists in determining the alternative that satisfies the fuzzy constraint C while attaining the fuzzy goal G (Bellman and Zadeh, 1970). The current operating objectives are thus considered as flexible constraints, i.e. constraints that can be partially satisfied (Dubois and Fortemps, 1999), while the fuzzy goal is obtained from the maximizing decisions calculated at the previous stage. The fuzzy goal obviously reflects the future consequences associated with a current decision, whereas the flexible constraints represent the immediate satisfaction of that decision.

When the decision maker is able to provide the relative importance of the future consequences and its evolution as a function of the time of the year, the aggregation of all objectives, i.e. the flexible constraints and the goal, can be achieved by a weighted sum. The fact that Yager's power raising method (Yager, 1978) is essentially pessimistic – the decision function is dominated by the less satisfied objectives – makes it inappropriate to form the decision function in an infinite-horizon FSDP algorithm. As matter of fact, the lack of compensation induced by the max-min formulation (or eventually the max-product formulation) implies that the FSDP algorithm might not converge. Other studies reported in the literature aggregate the objectives using a weighted sum, whose weighting coefficients are usually assigned by the decision maker or calculated, for example, by Saaty's method of pairwise comparisons (Saaty, 1980). See for example Bellman and Zadeh (1970) and Kacprzyk (1997). Only the flexible constraints are aggregated by a weighted sum by Fontane *et al.* (1997) and Tilmant *et al.* (2001). All of the reported studies implicitly assume that the aggregation is rather optimistic, i.e. the role played by the largest membership grades is more important than that of the smaller membership grades. As this paper demonstrates, this assumption may not yield to the best system performance. Careful attention should therefore be paid when aggregating unequal objectives with a weighted function to solve multistage decision-making problems with DP-based algorithms.

To illustrate this, a continuous FSDP model is developed and implemented to derive optimal operating policies for the Machadinho-Ita hydropower system in the Uruguay River basin in Brazil. Brazilian reservoirs are operated at the national scale by an independent coordinating agency. Machadinho and Ita reservoirs are used to illustrate the methodology and the influence of the rest of the Brazilian system is ignored. Rather, we assume that decision maker's preferences with regards to appropriate storage levels are available and can be represented by fuzzy sets. These fuzzy sets constitute flexible constraints that affect the operation of the reservoirs in conjunction with both hydropower generation and the future operations. This paper is organized as follows. Flexible stochastic dynamic programming (FSDP) and the reservoir operation problem are discussed first. Then, the most common techniques for including unequal objectives are presented. The application of the FSDP algorithm to the case study and the optimization results are discussed next. Finally, a summary and concluding remarks are given.

2. Flexible Stochastic Dynamic Programming

The optimal operation of a multipurpose reservoir is a multiobjective multistage optimization problem whose dynamics is described by the continuity equation of that reservoir:

$$S_{t+1} = S_t + Q_t - R_t - e_t \tag{1}$$

where t is the index of time period; S_t is the volume in storage at the beginning of period t; Q_t is the total inflow to the reservoir during time period t; R_t is the release decision during period t, and e_t are the evaporation losses during period t.

In classical optimization approaches, the performance of the reservoir resource is usually described through a benefit or loss function, which expresses the economic gains or losses associated with a release decision. Although useful, these approaches present limitations when non-economic objectives have to be taken into account. For example, water resources projects have an impact on a variety of economic, political, social and environmental objectives, whose conversion to monetary terms is still a matter of debate amongst economists (Shabman and Stephenson, 2000). In the FSDP algorithm, the performance of the reservoir is evaluated by K unequal objectives $O_t = \{O_{1,t}, O_{2,t}, \ldots, O_{K,t}\}$, which reflect the various concerns such as the degrees to which current and future operating objectives (services) are satisfied or affected by the current release decision R_t. These services are expressed here through linguistic variables and mathematically represented by fuzzy sets, whose membership functions are derived from surveys of water users and managers.

In the FSDP algorithm, the first K-1 services may typically include the immediate consequences associated with hydropower generation, irrigation, navigation, recreation, environmental considerations, etc., while the K^{th} objective ensure continuing satisfactory operation, therefore representing the future consequences of the release decision. Let $\mu_{o_i,t}(S_t, Q_t, R_t) \in [0, 1]$ be the satisfaction degree of objective $O_{i,t}$ by the alternative (S_t, Q_t, R_t). Assume $\omega_{i,t}$ is a non-negative number indicative of the relative importance of the objective $O_{i,t}$ during time period t. Assume f a function for including the relative importance of each objective. Then, at stage n (time t), the evaluation vector $\boldsymbol{O}_t \in \mathbb{R}^{K\times 1}$ is:

$$\boldsymbol{O}_t = [\mu^{eff}_{O_1,t}(.)\mu^{eff}_{O_2,t}(.)\ldots\mu^{eff}_{O_K,t}(.)]^\tau$$

where $\mu^{eff}_{O_i,t} = f(\mu_{O_i,t}(.)\omega_{i,t})$ is the effective satisfaction of objective $O_{i,t}$ during period t, and τ is the transpose operation.

The fuzzy decision set D_t results from the aggregation $\underbrace{\odot : [0,1] \times [0,1] \times \ldots \times [0,1]}_{K\ times} \rightarrow [0,1]$ of the K objectives $O_{i,t}$, $i = 1, 2, \ldots, K$. This aggregation corresponds to some operation on K fuzzy sets, and more specifically on the K membership functions $\mu^{eff}_{O_i,t}(.)$. As mentioned earlier, the interpretation of the aggregation is generally left to the decision maker, and can therefore be tailored to the decision-making problem.

Recall that the classical reservoir operation problem is often defined as the determination of a sequence of T release decisions such that the expected economic value of the system is maximized. In a fuzzy environment, the expected effective satisfaction associated with the operation of the system is the performance indicator that guides the multistage decision-making process:

$$E\left[\mu_{D_1}(S_1, Q_1, R_1) \odot \mu_{D_2}(S_2, Q_2, R_2) \odot \ldots \odot \mu_{D_T}(S_T, Q_T, R_T)\right] \tag{2}$$

with

$$\mu_{D_t}(.) = \mu^{eff}_{O_1,t}(.) \odot \mu^{eff}_{O_2,t}(.) \odot \ldots \odot \mu^{eff}_{O_K,t}(.)(S_{t+1}) \qquad t = 1, 2 \ldots T$$

$$\mu^{eff}_{O_K,T}(S_{T+1}) = f(\mu_G(S_{T+1}), \omega_{K,T})$$

where $\mu_G(S_{T+1})$ is a specified fuzzy goal at termination time T, and $\odot$ can be any suitable aggregation operator.

The recursive solution of the fuzzy DP (FDP) equation can be used to solve the fuzzy multistage decision-making problem (2)

$$\mu^*_{G_n}(S_t) = E_{Q_t}\left[\bigvee\nolimits_{R_t}\left\{\mu^{eff}_{O_1,t}(S_t, Q_t, R_t) \odot \mu^{eff}_{O_2,t}(S_t, Q_t, R_t) \odot \ldots \odot \mu^{eff}_{O_K,t}(S_{t+1})\right\}\right] \tag{3}$$

with $\mu^{eff}_{O_K,t}(S_{t+1}) = f(\mu^*_{G_{n-1}}(S_{t+1}), \omega_{K,t})$ where n = number of stages remaining until the end of the planning horizon; t = index of period; S_t = storage at the beginning of period t; Q_t = inflow during period t; R_t = release during period t; $\mu^*_{G_n}(S_t)$ = expected membership grade from the optimal operation of the system from the current period t to the end of the planning horizon given that the system's status in period t is S_t; $\mu^{eff}_{O_i,t}(S_{t+1})$ = effective membership grade of the i^{th} objective; E = expectation operator; $\odot$ = aggregation operator; $\vee$ = maximum operator.

As in classical DP, the functional FDP equation can be extended so as to incorporate the temporal persistence of the hydrologic conditions through the use of a hydrologic state variable H_t (Figure 1). The optimal operating policy can therefore be found by the recursive solution of a general fuzzy stochastic dynamic program (FSDP):

$$\mu^*_{G_n}(S_t, H_t) = E_{Q_t|H_t}[\bigvee_{R_t}\{\mu^{eff}_{O_1,t}(S_t, Q_t, R_t) \odot \mu^{eff}_{O_2,t}(S_t, Q_t, R_t) \odot \ldots$$

$$\mu^{eff}_{O_{K-1},t}(S_t, Q_t, R_t) \odot E_{H_{t+1}}|Q_t \mu^{eff}_{O_K,t}(S_{t+1}, H_{t+1})\}] \tag{4}$$

subject to

$$\max\left[R_{\min}, S_t + Q_t - S_{\max}\right] \leq R_t \leq \min\left[R_{\max}, S_t + Q_t - S_{\min}\right]$$

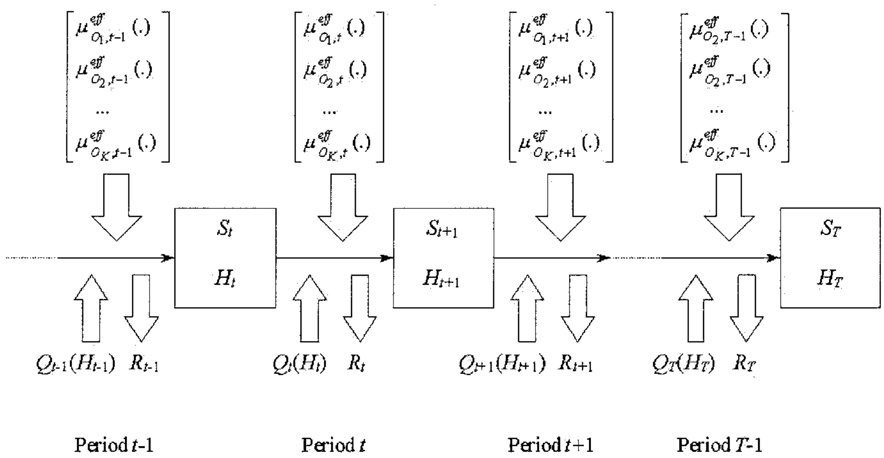

Figure 1. Reservoir Operation in a Stochastic and Fuzzy Environment.

where

H_t	hydrologic state in period t;
$\mu^*_{G_n}(S_t, H_t)$	expected membership grade from the optimal operation of the system from the current period t to the end of the planning horizon given that the system's status in period t is (S_t, it H$_t$);
$\mu^{eff}_{O_K,t}(S_{t+1}, H_{t+1})$	membership grade of the K^{th} objective = fuzzy goal† = effective satisfaction degree associated with the optimal operation of the system from the next period t+1 to the end of the planning horizon given that the system's status in period t+1 is $(S_{t+1}, \mathrm{H}_{t+1}) = f(\mu^*_{G_{n-1}}(S_{t+1}, H_{t+1}), \omega_{K,t})$
$\mu^{eff}_{O_1,t}(S_t, Q_t, R_t)$	membership grade of the i^{th} objective = i^{th} flexible constraint = effective satisfaction of objective $O_{i,t}$ during period t given the alternative (S_t, Q_t, R_t);

An important issue associated with discrete DP-based algorithms is the computational challenge posed by the solution of the functional DP equation for multidimensional problems. This phenomenon is popularly known as the 'curse of dimensionality'. Consider, for example, a classical DP problem with D state variables (dimensions), and where each variable is represented by a vector of N points. Then the grid would contain N^D points. At each stage, the functional DP equation is evaluated at each grid point so that the computational effort is proportional to N^D and to the work $W(P, D)$ required to solve the optimization problem, where P is the dimension of the decision vector. In fuzzy DP, the work W also depends on the selected aggregation operator $\odot$. If the most common aggregation operators, such as

† In this study, we use the term objective to include both the goal and the constraints

the min, the max, the product, the sum, are not computationally demanding, others may become prohibitive and seriously limit the applicability of the technique. The numerical efficiency is therefore one of the criteria for selecting the aggregation operator (Zimmermann, 1991).

3. Aggregation of Unequal Objectives in FSDP

Several functions for including weight factors in conjunction with different aggregation operators are available in the literature to derive suitable decision functions. Yager (1978) combines the product and min-operators ($\wedge$) with power of importance. The decision function becomes:

$$D_t(.) = \prod_{i=1}^{K} \{\mu_{O_i,t}(.)\}^{\omega_{i,t}} \tag{5}$$

or

$$D_t(.) = \bigwedge_{i=1}^{K} \{\mu_{O_i,t}(.)\}^{\omega_{i,t}} \tag{6}$$

The rationale behind Yager's approach is that when a weight factor is large, the effective membership grade becomes smaller, and therefore increases its importance in the min or in the product operations. Nevertheless, as pointed out by Kaymak and van Nauta Lemke (1998), this approach is suitable for pessimistic decision maker only.

Another common method for including relative importance to form a suitable decision function is the weighted function in which the satisfaction degrees $\mu_{O_i,t}(.)$ are directly multiplied by their weighting coefficients $\omega_{i,t}$:

$$D_t(.) = \sum_{i=1}^{K} \omega_{i,t}\mu_{O_i,t}(.) \tag{7}$$

with $\sum_{i=1}^{K} \omega_{i,t} = 1$

This weighted function has been adopted in many studies. See for example Fontane *et al.* (1997), Tilmant *et al.* (2001), Kacprzyk (1997), Esogbue and Kacprzyk (1998). If all of these studies use the weighted sum to aggregate unequal objectives, they pay little attention to the other properties of this operator. For example, (7) is known for leading to too optimistic results (Kaymak and van Nauta Lemke, 1998), which makes it suitable to model optimistic decision makers, i.e. decision makers that selects risky solutions. The same authors also discuss the conditions to be met for introducing weighting coefficients into the decision function, and conclude that the following general weighted function is a suitable decision function:

$$D_t(.) = \left[\sum_{i=1}^{K} \omega_{i,t}\mu_{O_i,t}(.)^s\right]^{1/s}, s \in \mathbb{R} \setminus \{0\} \tag{8}$$

$$D_t(.) = \prod_{i=1}^{K} \mu_{O_i,t}(.)^{\omega_{i,t}}, s = 0 \tag{9}$$

The value of the parameter s, also called the optimism index, can be modified to adjust the meaning of the aggregation. For positive value of s, the influence of the objectives that are best satisfied (high membership grade) increases in the decision function. For negative value of s, the decision function is more determined by the objectives that are less satisfied (low membership grade), and the decision-making becomes pessimistic. In most of the reported studies, the meaning of the parameter s is not discussed nor investigated and is systematically set equal to one. In this paper, we demonstrate that this systematic assumption may not be appropriate to solve decision-making problems, and water resource-based problems in particular. We illustrate this with a multireservoir system whose main objectives are the production of electricity and flood control. The reservoir operation problem is solved by an FSDP algorithm based on equation (4) and on the aggregation functions (8)–(9):

$$\mu^*_{G_n}(S_t, H_t) = E_{Q_t|H_t}\left[\bigvee_{R_t}\left\{\sum_{i=1}^{K-1}\omega_{i,t}\mu_{o_i,t}(S_t, Q_t, R_t)^s + \omega_{K,t}(E_{H_{t+1}|H_t,Q_t}\mu^*_{G_{n-1}}(S_{t+1}, H_{t+1}))^s\right\}^{1/s}\right] \tag{10}$$

FSDP-derived policies are then used to simulate the system using historical flows records. Simulation results are further compared to examine the influence of the s parameter on the performance of the system. We show that the conventional optimistic behavior ($s = 1$) may not yield the best system performance, especially with regards to the secondary objective. More specifically, the optimistic behavior tends to favor aggressive release policies, therefore increasing the risk of future restrictions during the low flow season. A best compromise solution between energy generation and the need to ensure continuing satisfactory operation can be achieved with slightly more pessimistic decision-making.

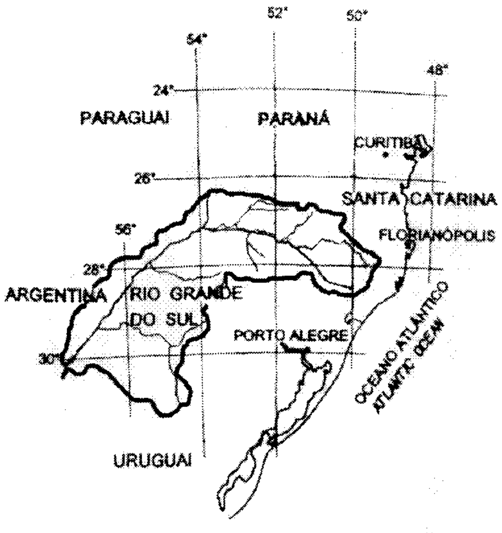

Figure 2. Location of the Uruguay River Basin.

Table I. Key features of Machadinho and Ita Dams

Characteristics	Machadinho	Ita
Drainage Area [km^2]	32050	44500
Storage Capacity [10^9 m^3]	3.34	5.10
Usable Storage Capacity [10^9m^3]	1.04	0.79
Normal Head [m]	105.2	104.9
Plant Capability [MW]	1140	1450
Max. Release [m^3/s]	1305	1590

4. The Hydroelectric Complex of the Uruguay River Basin

4.1. GENERAL FEATURES

The Uruguay River drains a basin of approximately 73000 km^2 located in the Southern Brazilian Highlands, at the border between Rio Grande do Sul and Santa Catarina States. It runs from East to West for around 850 km, till the Argentinean border (Figure 2). The basin area is essentially an agricultural region with a coastal mountain chain, called the 'Serra do Mar', in its upstream part.

The early studies of the hydroelectric development of the Uruguay River basin date from the Brazilian Hydro-energetic Inventory Studies of the late 60ies. The hydroelectric development scheme consists of a cascade of reservoirs from elevation 940 m to elevation 165 m, with an expected potential of 9700 MW. In the upstream part of the river basin, the projects that are under construction or constructed are

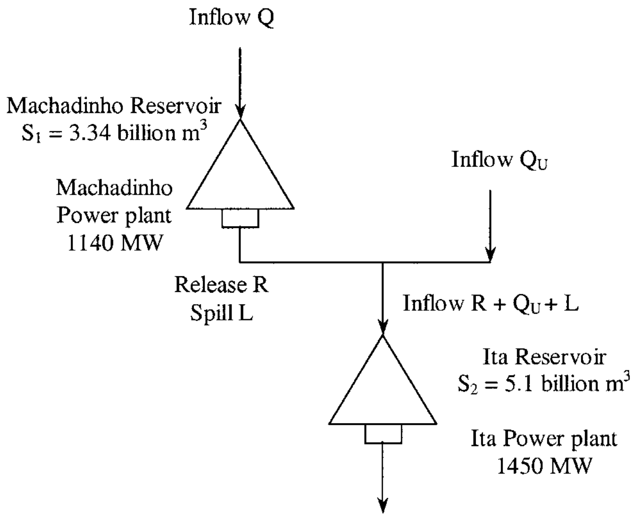

Figure 3. Schematic Representation of the Machadinho-Ita System (MIS).

the Machadinho hydropower plant on the Pelotas river, and the Ita hydropower plant on the Uruguay river, downstream of the confluence between the Pelotas and Do Peixe rivers. Key features of both reservoirs are given in Table I. Because of the conjunction of a severe financial crisis faced by the Brazilian State in the early 90ies and the growing demand for energy, both projects were carried out with the financial and the technical resources of private companies.

Both reservoirs will be connected to the Brazilian electrical system, which is characterized by a substantial percentage of hydro generation (more than 95% of the installed capacity). Details on the optimal operation of the Brazilian hydro-thermal system are found in Pereira (1989), Pereira (1998), and Cepel (1999).

4.2. HYDROLOGIC CHARACTERISTICS

The Machadinho-Ita hydroelectric system consists of two hydropower dams located along the Uruguay River. Two natural inflows enter the system: one major inflow Q to the Machadinho reservoir, and one lateral inflow Q_U. At any time, the total inflow to Ita reservoir is thus the sum of the side inflow Q_U, the release R and spill L from Machadinho. This system is pictured in Figure 3.

Historical monthly inflow records from 1931 to 1994 are used to derive the stochastic properties of the hydrologic input. Because the major inflow Q and the lateral inflow Q_U are strongly correlated ($\rho_{Q,Q_U} = 0.93$, where ρ is the correlation coefficient), the statistical analysis is carried out on the major inflows only. Chi-square goodness of fit as well as graphic tests reveal that each monthly inflow can be considered as a random variable with log-normal distribution. Another important assumption is made: the generating process is assumed periodically stationary.

5. FSDP Model of the Machadinho-Ita System (MIS)

Machadinho and Ita hydropower plants are the first major units of the hydroelectric development of the Uruguay River basin. The production of energy is therefore the main operating objective with an installed capacity of 2600 MW. Flood control is an important secondary objective, especially during the high flow season, from May to October. An adequate flood control strategy should yield flood mitigation without significantly affecting revenues from hydropower generation. Although important from a political point-of-view, maintaining minimum release for environmental and sanitation considerations does not constitute here a restricting constraint, and is therefore not taken into account in the optimization process. The operating constraints are thus limited to two: hydroelectric production and flood control.

Hydropower production and flood control are two conflicting services since flood control requires free volume in the reservoir, whereas hydropower generation is directly proportional to the storage level. Another practical issue comes from the fact that flood damage-discharge curves are extremely difficult to establish due to the lack of data, and the inherent imprecision associated with the estimation of flood damages (Bárdossy *et al.*, 1991). The vagueness of the flood control objective, as well as the fact that trade-offs between conflicting objectives have to be found, make the use of a flexible optimization technique attractive for solving the reservoir operation problem.

5.1. ASSUMPTIONS

Because the Machadinho reservoir is located immediately upstream of Ita, and since its usable storage capacity dominates that of Ita, the operation of the MIS can be considered as a single reservoir problem. Under this assumption, Ita is operated so as to keep the storage level constant. The single control variable is thus the monthly release from Machadinho. The inflow to the Ita reservoir includes the release through the turbines of Machadinho hydropower plant, the spill from Machadinho, and the inflows from uncontrolled tributaries. There is a strong correlation between the inflows to the Machadinho reservoir and the uncontrolled tributaries. Consequently, the state variables are the volume in storage at the beginning of a month in Machadinho reservoir, whereas the hydrologic condition is represented by the current inflow to the Machadinho reservoir. It is also assumed that the outflow from Machadinho is immediately available to Ita. With the previous assumptions, the FSDP model for the MIS can be written as follows:

Table II. Definition of variables and parameters for the FSDP Model for MIS

State variable – Machadinho reservoir	
Q_t	characteristic flows, unit = [10^6m^3];
S_t	characteristic storage volumes, unit = [10^6m^3];
S_{min}	dead storage capacity, unit = [10^6m^3];
S_{max}	maximum storage capacity, unit = [10^6m^3];
Decision variable – Machadinho reservoir	
R_t	release during month t, units = [10^6m^3];
R_{min}	minimum release, unit = [10^6m^3];
R_{max}	maximum turbined release, unit = [10^6m^3];
Fuzzy objectives	
$\mu_{O_{1,t}}(S_t, Q_t, R_t$	membership function for *efficient hydropower generation* in month t – Machadinho reservoir, unit = [–];
$\mu_{O_{2,t}}(S_t, Q_t, R_t$	membership function for *adequate storage level* in month t – Machadinho reservoir, unit = [–];
$\mu_{O_{3,t}}(S_{1,t}, Q_{1,t}, R_{1,t}$	membership function for *efficient hydropower generation* in month t – Ita reservoir, unit = [–];
$\mu^*_{G_{n-1}}(S_{t+1}, Q_{t+1}, R_{t+1}$	membership function for *suitable future operations*, from *t+1* to the end of the planning horizon– entire system, unit = [–];
Aggregation parameters	
$\omega_{1,t}$	weighting coefficient associated to hydropower – Machadinho reservoir, unit = [–];
$\omega_{2,t}$	weighting coefficient associated to the control of the storage level – Machadinho reservoir, unit = [–]
$\omega_{3,t}$	weighting coefficient associated to hydropower – Ita reservoir, unit = [–];
$\omega_{4,t}$	weighting coefficient associated to future operations – entire system, unit = [0-] ;
s	optimism index, $s \in \mathbb{R}$, unit = [–];
Hydrologic parameters	
$\rho(.)$	flow transition probability, unit = [–];
Additional variables and parameters	
L_t	spillage losses from the Machadinho reservoir, unit = [10^6 m^3];
$S_{I,t}$	target storage volume – Ita reservoir, unit = [10^6 m^3];
$Q_{I,t}$	total inflow to the Ita reservoir, unit = [10^6m^3];
$R_{I,t}$	turbined release from the Ita reservoir, unit = [10^6m^3];
$R_{I,max}$	maximum turbined release – Ita reservoir, unit = [10^6m^3];
$Q_{U,t}$	inflow from uncontrolled tributaries, unit = [10^6m^3];

$$\mu^*_{G_n}(S_t, Q_t) = E_{Q_t|H_t}\left[\max_{R_t}\left\{\sum_{i=1}^{2}\omega_{i,t}\mu_{O_i,t}(S_t, Q_t, R_t)^s + \right.\right.$$

$$\omega_{3,t}\mu_{O_{3},t}(S_{l,t}, Q_{l,t}, R_{l,t})^s +$$

$$\left.\left.\omega_{4,t}\left(\sum_{Q_t+1}\rho(Q_{t+1}|Q_t)\mu^*_{G_{n-1}}(S_{t+1}, Q_{t+1})\right)^s\right\}^{1/s}\right] \tag{11}$$

subject to

$$\max[R_{\min}, S_t + Q_t - S_{\max}] \leq R_t \leq \min[R_{\max}, S_t + Q_t - S_{\min}]$$

$$S_{t+1} = S_t + Q_t - R_t - L_t \tag{12}$$

where

$$Q_{I,t} = R_t + L_t + Q_{U,t}(Q_t)$$

$$R_{I,t} = \min[Q_{I,t}, R_{I,\max}]$$

Variables and parameters are listed in Table II.

5.2. CONVERGENCE

The FSDP model (11) is run iteratively until it generates steady-state solutions, i.e. optimal release policy tables and membership functions. The convergence criterion is evaluated every year in period 1 (January), and consists in checking that the difference between the current membership function and the membership function from the previous cycle becomes negligible for all grid points (S_t, Q_t). In this study, the FSDP model stops as soon as $|\mu^*_{G,c-1} - \mu^*_{G,c}| \leq 0.001$, where c is the number of iterations.

Several FSDP models (11) with different values for the optimism index s are developed and implemented to solve the reservoir operation problem. Note that the model (11) is run with $s \in [-20,1]$. The lower bound is chosen because the model (11) does not converge for $s < -20$, whereas the upper bound corresponds to the traditional (optimistic) averaging operator. It would be inconsistent from a logical point-of-view to consider $s > 1$ since the aggregation operator (8) would become too optimistic, since paying too much attention to the most achieved objective to the detriment of the least achieved ones. In other words, the aggregation would then be close to the logical ‘or’, which cannot be retained here for equity consideration since the decision maker usually wants to satisfy all objectives. The lack of convergence of the FSDP model when too pessimistic decisions are taken

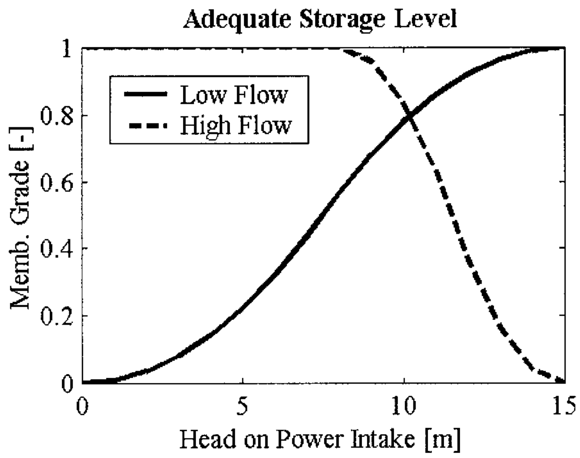

Figure 4. Example of membership functions.

($s \ll 0$) results of the so-called *Drowning Effect* (Dubois *et al.*, 1995; Dubois and Fortemps, 1999). By devoting significant attention to the least achieved objective, the decision-making becomes independent of the satisfaction degrees of the other objectives. As the number of iteration n increases, the membership grades of the K^{th} objective (the goal) keeps decreasing due to the recursive solution of (11) and the pessimistic aggregation, which becomes close to the min-operator. The satisfaction degrees of the first K-1 objectives, the flexible constraints, are therefore not considered so that the model cannot balance the immediate and the future consequences.

5.3. ASSESSMENT OF MEMBERSHIP FUNCTIONS AND WEIGHT FACTORS

In the FSDP formulation (9), operating objectives are considered as flexible constraints, and mathematically encoded as fuzzy sets. A reliable definition of the membership functions associated with the operating objectives is a crucial point since the machinery of fuzzy mathematics directly relies on the membership functions of the fuzzy quantities.

During the high flow season, a portion of the reservoir is dedicated to emptiness so that excess inflow can be stored and gradually released at lower rates. In terms of flexible constraint, the membership function associated to this objective is constructed so that low storage levels are encouraged, while high levels must be avoided. Between these two extremes are situations in which medium pool elevations are tolerated. The opposite behavior is expected during the low flow season. In the absence of major flooding events, high pool elevations are preferred during the summer because the efficiency of the hydropower plant increases with the elevation. In brief, the control of the storage level consists in encouraging low

Table III. Weight factors

Season	Hydropower (Machadinho)	Storage Control (Machadinho)	Hydropower (Ita)	Goal (MIS)
Low Flow				
Nov. – May	0.21	0.09	0.49	0.21
High Flow				
Jun. – Oct.	0.06	0.41	0.12	0.41

levels during the wet season, and high levels during the rest of the year. These membership functions are pictured in Figure 4.

The membership function characterizing 'efficient' hydropower generation is based on the assumption that the satisfaction is proportional to the ratio between current and maximum allowable production. For example, if R_t is the release through the turbines of Machadinho hydropower plant, and $\bar{E}$ is the average effective head of that reservoir, the satisfaction degree is given by:

$$\mu_{o_{1,t}}(.) = \frac{R_t \bar{E}_t \varepsilon(R_t, S_t)}{R_{\max} E_{\max} \varepsilon(R_{\max}, S_{\max}} \tag{13}$$

where $\varepsilon(.)$ is the plant efficiency.

Also of particular importance in this study is the determination of the weighting coefficients $\not{p}_{i,t}$. These coefficients reflect the relative importance of the associated objective in the decision function. Choo *et al.* (1999) provide a timely review of the interpretation of criteria weights in multicriteria decision-making. Here, the weighting coefficient ω_i must be interpreted as the relative importance of the objective O_i with respect to the overall goal. The question posed to the decision maker is: 'Of the two objectives O_i and O_j being compared, which one is considered more important, and by how many times with respect to the overall goal?' Weighting coefficients are assumed to be 'crisp' numbers and their determination relies on Saaty's method of pairwise comparisons (Saaty, 1980). Two sets of weight factors are calculated and listed in Table III. The first set is specific to the high flow season. During that period, the flood control objective plays a major role. Future consequences also receive particular attention in order to preserve the resource so that future hydropower generation is not compromised. During the low flow season, the priority shifts to the production of energy from the two hydropower plants because energy prices tend to increase when the hydrological inputs decrease. The reason is found in the increasing marginal costs of the hydrothermal electrical system when thermal plants must substitute hydroelectrical ones.

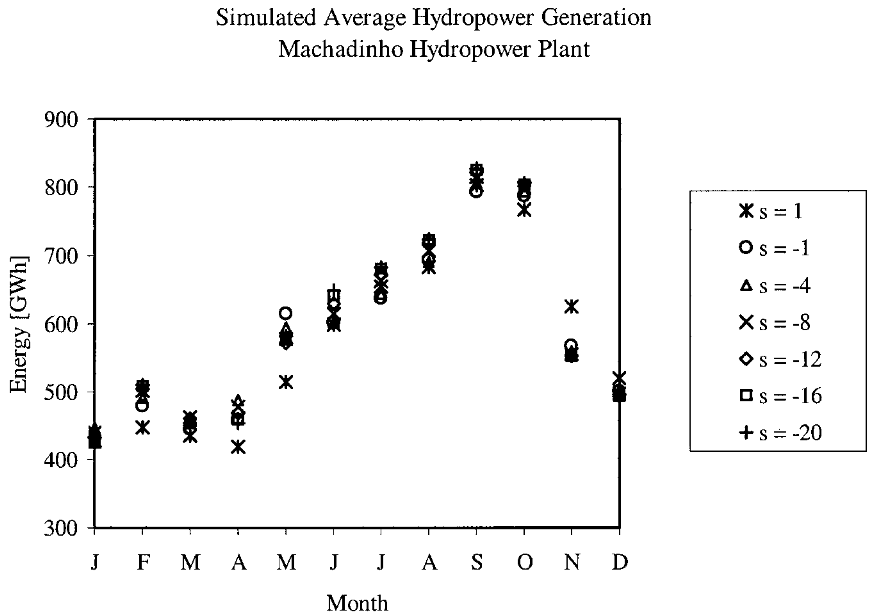

Figure 5. Simulated Average Hydropower Generation – Machadinho Hydropower Plant.

6. Simulation of FSDP results

System performance is traditionally estimated from simulation analysis using historical and/or synthetic flows. Here, historical monthly inflows to Machadinho as well as lateral flows, from 1931 to 1994, are used for simulating the MIS. This 64-year long time series of monthly flows is assumed representative of the stochastic process. Simulation results consist of the end-of-the month storage levels in both reservoirs, the monthly release through the turbines of each plant, the spillage losses for both reservoirs, and the average monthly hydroelectric production of each plant. These results are obtained after implementing a reoptimization model with the continuity equation (12). See appendix 1 for further details on the reoptimization approach in FSDP. Each simulation run starts with initial storages of 3000 million m^3 for the Machadinho reservoir and 5000 million m^3 for the Ita reservoir.

Figure 5 displays the monthly average simulated hydropower generation for the Machadinho hydropower plant. These results are given for different indices of optimism: $s \in \{1, -1, -4, -8, -12, -16, -20\}$. It can be seen that the parameter s has little effect on the average energy production of this reservoir. Due to the fairly small usable head (about 15% of the total head), seasonal changes in energy production are more pronounced than those induced by different decision-making behaviors. Hence, more conservative release polices seem to have little long-term impact on hydropower generation. Consider, for example, the low flow season. In hydro-dominated electrical system, it is often beneficial to produce energy during the low flow season because marginal costs are higher due to the substitution of

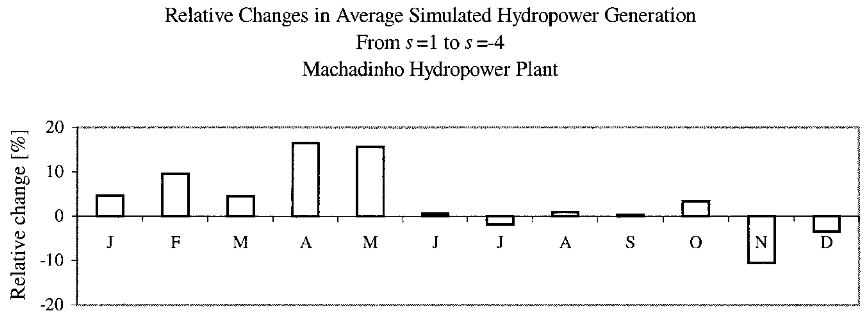

Figure 6. Relative Changes in Energy Production.

hydro by thermal power plants. For this case study, moving from optimistic to pessimistic decision-making implies that more attention is paid to the objectives that encourage high storage levels, such as the fuzzy goal and the control of the storage level in the Machadinho reservoir (see Table III). Higher heads on turbines therefore compensate the reduction in releases, but also increase the possibilities of future releases, therefore contributing to a slight improvement in energy production during this season. For example, when the optimism index s moves from 1 to –4, the average annual production increases by 2.5% from 6897 to 7073 GWh. Nevertheless, the analysis of the average monthly productions reveals that the relative positive changes in energy production essentially occur during the low flow season (from Nov. to May) when energy prices are expected to be higher. This is shown on Figure 6, which displays the relative changes in energy production when the optimism index s moves from 1 to –4. The higher production levels observed at the beginning of the dry season (Nov. – Dec.) when $s = 1$ result from the adjustment of the system when moving from aggressive release policies that characterize the wet season to the more conservative release policies of the dry season. From January to May, however, hydropower increases by as much as 15% with $s = -4$ resulting in increased hydro revenues (by at least 15% depending on the marginal cost-inflow relationship of this system).

Also of particular importance is the relationship between s, the release R_t, and the hydrological regime. As s is getting lower, the policy is expected to be more conservative, i.e. the release decisions are not too risky. This mechanism should be accentuated during the low flow season. As a matter of fact, since the transition probabilities indicate that low flows tend to be followed by low flows, a pessimistic decision maker should become even more pessimistic under low flow conditions. Figure 7 displays 'pessimistic' releases versus 'optimistic' releases for two different months representative of the two different seasons: the dry (Figure 7a) and the wet (Figure 7b) seasons. For both months, it can be seen that the interaction between s and R_t is a little bit more complex than expected. In fact, as long as the release does not exceed some limit, which seems to be around 800 m^3/s, the system is indeed more aggressive with $s = 1$. Note that this is even more

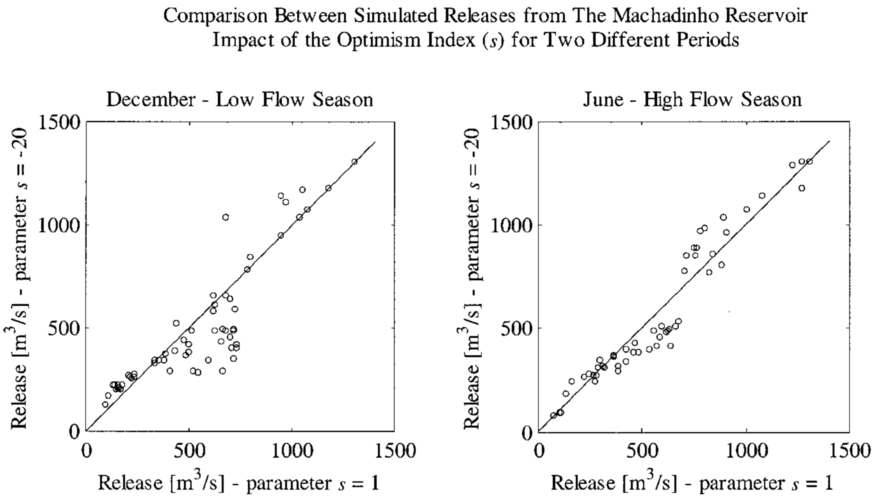

Figure 7. Illustration of 'Pessimistic' versus 'Optimistic' Releases for the Two Seasons.

pronounced during the dry season. But, as we can see, this does not become true under high flow conditions. The reason is that the 'pessimistic' policy (s = – 20) yields high storage levels with little flood control capability. Hence, under high flow conditions, 'pessimistic' releases may be larger than 'optimistic' ones because of the reduced possibility to catch floods.

As we just saw, the s parameter seems to have a major impact on the capability to capture flood waters during the winter and to maintain high pool level during the summer. If the decision maker selects too risky release decisions, low storage levels should be maintained throughout the year. As mentioned earlier, this might not constitute an interesting strategy during the summer. As a matter of fact, in order to achieve a certain amount of energy, more water should be released to compensate the lower head on the turbines. The opposite should be observed with a pessimistic decision maker. So, when it comes to maintaining adequate storage level, it seems that the best behavior should lie somewhere between the most pessimistic and the most optimistic bounds. This is confirmed by the examination of Figure 8, which presents the monthly evolution of the average satisfaction associated to the storage level for different values of s.

As we can see from Figure 8, the optimism index s has a major impact on the average volume in storage and thus on the associated membership grade. It is also obvious that the conventional averaging operator ($s = 1$) does not yield to the 'best' strategy for the control of the storage level since the average satisfaction is the lowest during the dry season, from November to April. Rather, a slightly more pessimistic behavior, with $s \in [-1, -4]$, generates more acceptable results with membership grades ranging from 0.6 to 0.8. Note that too pessimistic decisions have a negative impact on the storage level during the wet season, and are thus incompatible with a dependable flood control strategy.

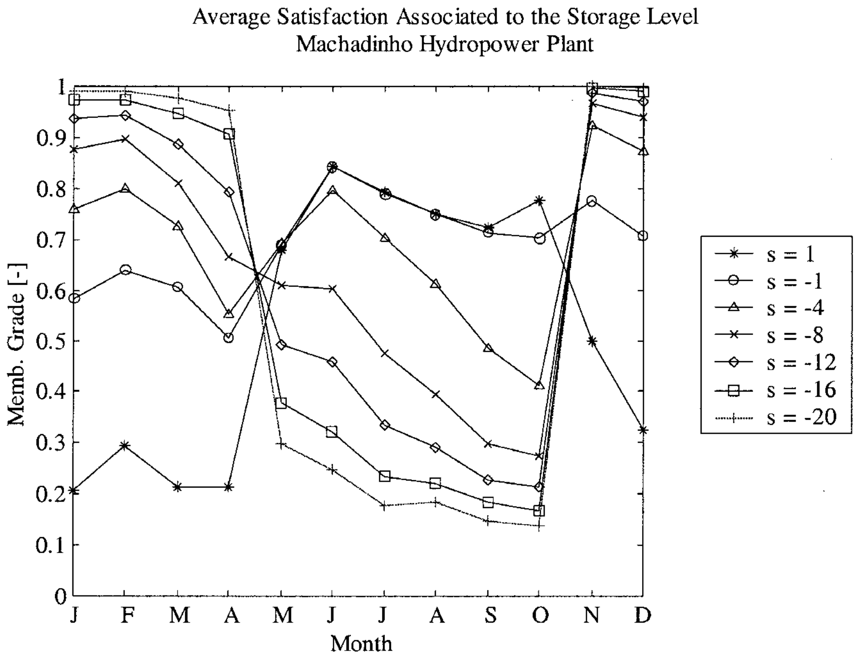

Figure 8. Simulated Average Satisfaction Associated to the Storage Level.

7. Summary and Conclusions

This paper presents a continuous FSDP algorithm to solve multipurpose multireservoir operation problems. Operating objectives are represented by fuzzy sets and aggregated by the generalized averaging operator, whose parameter can be interpreted as the decision maker index of optimism. This FSDP model is implemented to derive optimal operating policies for a hydropower system located in Brazil, where the main objectives are hydropower generation and flood control. Optimization results are then used to simulate the system with a continuous reoptimization approach.

This study also investigates the different meanings of the generalized averaging operator and their impacts on long-term performance of a reservoir resource. We show that the assumption that consists in using the conventional averaging operator, which corresponds to the generalized averaging operator with $s = 1$, may not yield to the 'best' performance. This is particularly true for time periods characterized by the need to preserve the resource. For example, such a need might be encountered during the dry season when future water deliveries (water supply, irrigation, recreation) strongly depend on the current release decisions. During that period, it is expected that solutions provided by the conventional averaging operator are inappropriate because they are too risky (optimistic). Rather, the decision maker should be more pessimistic, and thus pay more attention to the less effectively satisfied objectives.

Acknowledgments

The work described in this paper is supported by the FIRST-Université research program (Grant 981/3815). The authors sincerely thank Vicente Miranda Regina and Joao Miguel Martin (Gerasul – Centras Electricas do Sul do Brasil), as well as Christan Coen (Coyne e Bellier Engenharia) for their collaboration.

Appendix I. The Reoptimization Model

There are two approaches for implementing FSDP-derived solutions in actual or simulated operation, each depends on one of the two sets of solutions obtained from the FSDP model: either the optimal policy tables or the steady-state membership functions. The traditional method consists in interpolating directly in the FSDP tables $R_t^*(S_t, H_t)$ given the system's status $(S_{y,t}, H_{y,t})$, where $H_{y,t}$ is the current hydrologic state and $S_{y,t}$ is the storage at the beginning of time period t (year y). In the second approach, an optimization model is used to calculate the optimal release with a continuous objective function obtained from both $\mu^*_{G_{t+1}}(S_{t+1}, H_{t+1})$ and the system's status $({}_{y,t}, H_{y,t})$. Tejada-Guibert *et al.* (1993) show that the re-optimized SDP policy resulted in slightly better performance than the interpolated policy. In addition, the release tables generated by the FSDP algorithm may show discontinuities when the decision function is relatively flat. In the reoptimization approach, the optimal solution does not solely rely on the characteristic values employed in the FSDP algorithm. Rather, the approach uses continuous FSDP-derived membership functions and the exact system state to determine the optimal release:

$$\sum_{R_{y,t}}\{\mu^{eff}_{O_{1,t}}(S_{y,t}, Q_{y,t}, R_{y,t} \odot \mu^{eff}_{O_{2,t}}(S_{y,t}, Q_{y,t}, R_{y,t} \odot \ldots$$

$$\odot \mu^{eff}_{O_{K-1},t}(S_{y,t}, Q_{y,t}, R_{y,t}) \odot E_{H_{t+1}|H_{y,t}, Q^{tot}_{y,t}}$$

$$\mu^{eff}_{O_K,t}(S_{t+1}, H_{t+1})\}$$

where

t	index of time period;
y	index of year;
$\mathrm{S}_{y,t}$	Storage level at the beginning of period t, year y;
$H_{y,t}$	hydrologic state during period t, year y;
$Q_{y,t}$	vector of inflow to the reservoirs during period t, year y;
$Q_{y,t}^{tot}$	total inflow to the system, period t, year y;
$R_{y,t}$	vector of optimal releases during month t, year y;
H_{t+1}	vector of future characteristic hydrologic states, period t+1;
S_{t+1}	vector of future characteristic storage levels, period t+1;
$\mu_{O_K,t}^{eff}$	$= f(\mu_{G_{t+1}}^{ast}(S_{t+1}, H_{t+1}), \omega_{K,t})$

The evaluation of the reoptimization model requires the approximation of the discrete membership functions $\mu_{G_{t+1}}^{ast}(S_{t+1}, H_{t+1}$ by continuous functions. In this study, the approximation is carried out by cubic splines, i.e. individual multivariate cubic polynomials defined on each subregion of the state space domain delimited by the grid points (Johnson *et al.*, 1993).

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